FORM TP 2017141

TEST CODE **02105020**

MAY/JUNE 2017

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE sections. Answer ALL questions from the THREE sections.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
6. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer**.
7. You may use a silent, non-programmable calculator.

Examination Materials:

A list of mathematical formulae and tables (Revised 2012)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

SECTION A

MODULE 1: COLLECTING AND DESCRIBING DATA

Answer BOTH questions.

1. (a) Match the phrases or statement in Column **A** to the phrases in Column **B** by writing the number from Column A in the appropriate space in Column **C**.

	A	B	C
1	A sample	An example of quantitative data	
2	A sample survey	Data which can be counted	
3	The number of eggs collected this morning	A systematic random sample	
4	Discrete data	A method of collecting data from a sample	
5	Choose every tenth member of the population	A part of the population	

[5 marks]

(b) State THREE reasons why it may be necessary to take a sample when carrying out research.

.....

.....

.....

.....

.....

.....

.....

.....

.....

[3 marks]

(c) Determine whether EACH of the following refers to a sample or to a population.

- A group of 25 persons selected to test a new drug

  .....

- The total number of books sold at the book sale

  .....

[2 marks]

(d) State which of the following represents discrete data and which represents continuous data.

- Number of persons in a household

  .....

- Speed of a car in kilometres per hour

  .....

- Student enrolment in a college in the last year

  .....

[3 marks]

(e) The following table shows the type and number of workers employed by a garment factory. A sample of 15 workers must be selected to attend a conference.

Type of Worker	Number Employed
Cleaner	10
Cutter	20
Designer	5
Stitcher	25
Total	**60**

(i) Using a stratified random sample, calculate the number of workers from EACH group that will be in the sample.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[5 marks]

(ii) What is the advantage of using stratified random sampling rather than simple random sampling to select persons?

.....

.....

.....

.....

[2 marks]

(f) State the sampling method to be used in EACH of the following approaches to selecting a sample of 50 students from a school with a roll of 420 students distributed over 14 classes.

- Select a sample of 10 classes out of the 14 classes in the school, and then take a sample of 5 students from each class.

  .....

- Take a random sample of students from each class proportionate to the number of students in the class.

  .....

- Choose every 25th name on the school roll.

  .....

- Stand at the school gate and choose the first 50 students coming into the school.

  .....

- Put the names of all the students in a box and then randomly select 50 names from the box.

  .....

[5 marks]

Total 25 marks

2. (a) The following data are the marks that the 25 students in a class obtained in a statistics examination.

52	65	78	34	45	65	23	75	80	65	56	57
69	88	37	47	73	49	68	29	81	30	50	61
60											

(i) Using groups of 10 and starting at 20, construct a stem-and-leaf diagram to show the data.

[4 marks]

(ii) Use the stem-and-leaf diagram in (a) (i) on page 8 to determine the

mode

.....

[1 mark]

median

.....

[1 mark]

interquartile range

.....

.....

.....

.....

.....

.....

[3 marks]

number of students who got 60 marks or more.

.....

[1 mark]

(b) The number of days that 100 employees were late for work in the last year are displayed in the following table.

Number of Days Late	0–15	16–20	21–30	31–40	41–50	51–60	61–70	71–90
Number of Persons	3	5	16	24	26	14	10	2

(i) State ONE **disadvantage** of displaying the data in this form.

.....

.....

[1 mark]

(ii) State the boundaries of the SECOND class.

.....

[1 mark]

(iii) Calculate the frequency density of the FIFTH class.

.....

.....

[2 marks]

(iv) Showing your method clearly, determine the

- mean number of days that the employees were late

.....

.....

.....

.....

.....

.....

.....

.....

.....

[3 marks]

- standard deviation of the number of days employees were late

.....

.....

.....

.....

.....

.....

.....

.....

[4 marks]

- median class

.....

.....

.....

.....

[1 mark]

- median number of days employees were late.

.....

.....

.....

.....

.....

.....

.....

[3 marks]

Total 25 marks

SECTION B

MODULE 2: MANAGING UNCERTAINTY

Answer BOTH questions.

3. (a) Two events, M and N, are such that $P(M) = \frac{4}{5}$, $P(M \cup N) = \frac{7}{8}$ and $P(M \cap N) = \frac{1}{4}$.

(i) Calculate $P(N)$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

[3 marks]

(ii) State, with reason, whether

M and *N* are independent

.....

.....

.....

.....

.....

.....

.....

.....

[2 marks]

M and *N* are mutually exclusive.

.....

.....

.....

.....

[2 marks]

(b) Of the total monthly production of an item, 60% are manufactured for export and the rest are sold locally. Of the items produced for export, 3% are defective, and 4% of the items that are sold locally are defective.

(i) In the space below, draw a clearly labelled tree diagram to show this information.

[4 marks]

(ii) Calculate the probability that an item, selected at random, is defective.

.....

.....

.....

.....

.....

.....

.....

.....

.....

[3 marks]

(iii) Given that an item selected at random is defective, calculate the probability that it is an item for export. **Give your answer correct to three significant figures.**

.....

.....

.....

.....

.....

.....

.....

.....

.....

[3 marks]

(c) A random variable X has a probability distribution as shown in the following table.

X	0	1	2	3	4
$P(X = x)$	0.15	0.22	0.25	p	0.10

(i) Determine the value of p.

.....

.....

.....

.....

[2 marks]

(ii) Calculate

- $E(X)$

.....

.....

.....

.....

.....

.....

[3 marks]

- Var$[X]$.

.....

.....

.....

.....

.....

.....

[3 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) The 15 persons in a sports club play badminton or tennis. Ten persons play badminton and 8 persons play tennis. A person is selected at random from the sports club. Calculate the probability that the person plays

badminton

.....

.....

.....

[1 mark]

BOTH badminton and tennis

.....

.....

.....

.....

.....

.....

.....

[2 marks]

only one of the sports.

.....

.....

.....

.....

.....

.....

.....

[3 marks]

(b) The mass, X, of the packages of a certain type of breakfast cereal follows a normal distribution with a mean of 60 g and a standard deviation of 5 g. Calculate the probability that a package chosen at random has a mass of more than 68 g.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[5 marks]

(c) During an experiment, 8% of the vegetable seedlings produced will NOT survive transplantation. Fifteen such seedlings are transplanted. X is the number of seedlings that do NOT survive.

(i) State FOUR assumptions that are made in applying a binomial model to this experiment.

.....

.....

.....

.....

.....

.....

.....

.....

[4 marks]

(ii) State a binomial distribution that will model this experiment.

.....

.....

[1 mark]

(iii) What is the probability that exactly 4 seedlings will NOT survive transplantation?

.....

.....

.....

.....

[2 marks]

(iv) Five hundred seedlings are sent to KG's Gardens.

Determine the mean and the variance of the number of seedlings that will NOT survive transplantation.

.....

.....

.....

.....

[2 marks]

What is the probability that less than 55 seedlings will NOT survive transplantation?

.....

.....

.....

.....

.....

.....

[5 marks]

Total 25 marks

SECTION C

MODULE 3: ANALYSING AND INTERPRETING DATA

Answer BOTH questions.

5. (a) A random sample of 55 sweets was found to have a mean mass of 0.93 g and a standard deviation of 0.10 g. Determine an approximate 99% confidence interval for the mean mass of the sweets.

.....

.....

.....

.....

.....

.....

.....

.....

[4 marks]

(b) A certain supermarket found that the price codes on some packages were not clear. In a random sample of 80 packages, 18 had price codes that were not clear. Construct a 97% confidence interval for the proportion of packages whose price codes were not clear.

.....

.....

.....

.....

.....

.....

.....

.....

[4 marks]

GO ON TO THE NEXT PAGE

(c) A random variable X follows a normal distribution with mean, μ, and an unknown variance, σ^2. A random sample of 150 observations of X gave $\sum x = 1600$ and $\sum x^2 = 18\,040$. Calculate unbiased estimates for the mean and standard deviation of X.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[6 marks]

(d) Forty random samples of 5 soap bars are taken and a 90% confidence interval for the mean mass, μ, is calculated for EACH sample. Find the expected number of intervals that do NOT contain μ.

.....

.....

.....

.....

[2 marks]

GO ON TO THE NEXT PAGE

(e) The mean of 60 observations of X, where $X \sim \text{Bin}(10, 0.45)$ is $\overline{X}$.

(i) State the distribution of $\overline{X}$, giving its parameters.

.....

.....

.....

.....

.....

[4 marks]

(ii) Calculate $P(\overline{X} > 4.75)$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[5 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) The mass of bags of sugar packed by a machine follows a normal distribution with a mean mass of 250 g and a standard deviation of 5 g. The machine is adjusted after servicing but it is assumed that the standard deviation remains the same. A random sample of 50 bags taken from the machine after servicing gives a mean mass of 251.9 g. Test at the 5% level of significance whether the mean mass of the bags has changed.

(i) State clearly the null and alternate hypotheses.

.....

.....

.....

.....

[3 marks]

(ii) Identify the critical region(s) of the test statistic.

.....

.....

.....

.....

[4 marks]

(iii) Calculate the value of the test statistic.

.....

.....

.....

.....

[4 marks]

(iv) State clearly a valid conclusion of the test, giving a reason for your answer.

.....

.....

.....

.....

[3 marks]

GO ON TO THE NEXT PAGE

(b) A sample of 100 persons is asked whether children are given too much money to spend. The results of the survey are given in the following table.

Opinion	**Gender of Respondent**		**Total**
	Male	**Female**	
Agree	8	7	**15**
Have no opinion	26	24	**50**
Do not agree	7	28	**35**
Total	**41**	**59**	**100**

A chi-square test at the 5% level of significance is used to determine whether this sample presents sufficient evidence to reject the hypothesis that gender is independent of opinion.

(i) State clearly the null and alternative hypotheses.

.....

.....

.....

.....

[2 marks]

(ii) Calculate, to the **nearest whole number**, the expected number of males that have no opinion.

.....

.....

.....

.....

.....

.....

.....

.....

[3 marks]

GO ON TO THE NEXT PAGE

(iii) Identify the rejection region of the test.

.....

.....

.....

.....

[3 marks]

(iv) The calculated chi-square value of the test is 9.8249. State clearly the conclusion for this test, giving a reason for your answer.

.....

.....

.....

.....

.....

.....

[3 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.